Exam Program Correctness, April, 1st 2016, 9:00-12:00h.

- This exam consists of three problems. Problem 1 is worth 20 points, problem 2 is worth 30 points, and problem 3 is worth 40 points. You get 10 points for not misspelling your name and student number.
- Give complete annotations, and linear proofs. Use a pen. Do not use a pencil!
- The exam is a closed book exam. You are not allowed to use the reader, slides, notes, or any other material.
- Do not hand in scratch paper!

Problem 1 (20 pt). Declared are the variables $a,\ b,\ n : \mathbb{N}$. Design an annotated command S that satisfies the Hoare triple:

$$\{\ b \cdot a^n = X\ \wedge\ 2 \cdot Y \leq n < 2 \cdot (Y+1)\ \}\ \ S\ \ \{\ b \cdot a^n = X\ \wedge\ n = Y\ \}$$

You are not allowed to use a loop.

Problem 2 (30 pt). Design and prove the correctness of a command T that satisfies

const $n : \mathbb{Z},\ a :$ **array** $[0..n)$ **of** $\mathbb{Z}$;
var $z : \mathbb{Z}$;
  $\{\ P : n > 0\ \}$
T
  $\{\ Q : z = \mathsf{Max}\ (\mathsf{Min}\ (a[i] + a[j] \mid i,j : 0 \leq i \leq j \leq k) \mid k : 0 \leq k < n)\ \}$.

The time complexity of the command S must be linear in n. You are not allowed to use the values $\pm\infty$ in the program. Start by defining one or more suitable helper functions with corresponding recurrences.

Problem 3 (40 pt). Given is a two-dimensional array a that is *descending* in its first argument and *decreasing* in its second argument. Consider the following specification:

const $n,\ w : \mathbb{N},\ a :$ **array** $[0..n)$ **of** $\mathbb{N}$;
  var $z : \mathbb{N}$;
    $\{P :\ Z = \#\{(i,j) \mid i,j : 0 \leq i \leq j < n\ \wedge\ a[i,j] = w\}\ \}$
  U
    $\{Q :\ Z = z\}$

(a) Make a sketch in which you clearly indicate where the array is high, low, and how a contour line goes.

(b) Define a function $F(x,y)$ that can be used to compute Z. Determine the relevant recurrences for $F(x,y)$, including the base cases.

(c) Design a command U that has a linear time complexity in n. Prove the correctness of your solution. [Note: you can only receive points for part (c) if the recurrences in part (b) are correct.]